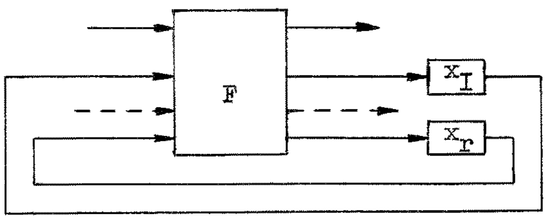
The equivalence of schemata with some feedbacks

M.Roytberg
Research Cumputer Center
Pushchino, Moscow Region,
U.S.S.R.

I. Introduction.

We study the schemata, their elements being automata. The definition of scheme of automata is given in [I]. In our work an automate means an initial Mealy automate.

I.I. Simple schemata. Every scheme of automata with r feedbacks may be represented in a form of so called simple scheme (see picture).



An automate F will be refered to as a basic automate of a simple scheme, automata $x_I,..,x_r$ - as feedback automata.

For convenience we'll consider only such schemata in which all feedback automata have the same input alphabets and the same output alphabets.

I.2. GM-automata. Definition. Let F be an automate with r+I input and r+I output, inputs and outputs of F are enumerated from 0 to r.

An automate F is a generalized Moore automate (GM-automate) if output **symbols** in all output channels except zero output channel depend only on the input symbol in zero channel and on the state of automate F. I.e. if F - GM-automate with the input alphabet $\Sigma_0 \times \Sigma^r$ and the output alphabet $T_0 \times T^r$, the set of states Q and the output function $\Psi: Q \times (\Sigma_0 \times \Sigma^r) \to T_0 \times T^r$, then

$$\forall q \in Q \forall \pi, \rho \in \Sigma_0 \times \Sigma^r \forall i \in \{1,\dots,r\} (\pi^0 = \rho^0 \supset \Psi^i(q,\pi) = \Psi^i(q,\rho)).$$

Mark, that the output symbol in zero output channel of GM-automate may depend on the input symbols in all input channels.

A number r is refered to as a dimension of a GM-automate F below. Dimension of GM-automate F is denoted dim F.

I.3. S-equivalence of GM-automata. It's evident, that if an automate F is a GM-automate with the input alphabet $\Sigma_0 \times \Sigma^r$ and the output alphabet $T_0 \times T^r$, then for any automata $x_I,..,x_r$ with the input alphabet T and the output alphabet Σ, the simple scheme with the basic automate F and the feedback automata $x_I,..,x_r$ is defined correctly.

An automate given by this scheme we'll denote as $S(F,x_I,..,x_r)$.

An input alphabet of $S(F,x_I,..,x_r)$ is Σ_o and an output alphabet is T_o.

Definition. Two r-dimensional GM-automata F and G are s-equivalent if for any automata $x_I,..,x_r$ the automata $S(F,x_I,..,x_r)$ and $S(F,x_I,..,x_r)$ are equivalent.

Definition. A word v and automata $x_I,..,x_r$ s-distinguish r-dimensional GM-automata F and G if the output words of automata $S(F,x_I,..,x_r)$ and $S(G,x_I,..,x_r)$ on the word v are different.

I.4. List of results. We give the algorithm which constructs a GM-automate with minimal number of states. This minimal GM-automate is unique in some exact sence (Theorems I,2).

Also we give an upper bound of a length of the experiment which determines s-equivalence of given GM-automata, i.e. a bound for the length of such word v, that v and some automata $x_I,..,x_r$ s-distinguish given GM-automata (Theorem 5). This upper bound can't be improved essentially (Theorem 6).

We generalise the concept of s-equivalence to the case of schemata with nonequal number of feedbacks and prove theorems analogous to above theorems (Theorems 3,4).

A problem of s-equivalence of two GM-automata is solvable becouse of above bounds.

2. The minimization of GM-automata.

Designation $F=\langle\Gamma,Q,\Delta,q_o,\varphi,\psi\rangle$ means Fis the automate with the input alphabet Γ, the set of states Q, the output alphabet Δ, the initial state q_o, the transition function $\varphi:Q\times\Gamma\rightarrow Q$, and the output function $\psi:Q\times\Gamma\rightarrow\Delta$.

2.I. The normal form of the GM-automate. Definition. Let F be a GM-automate with the input alphabet $\Sigma_o\times\Sigma^r$ and the output alphabet $T_o\times T^r$, q is a state of F and $\sigma_o\in\Sigma_o$.

A feedback channel i ($i\in\{I,..,r\}$) is named q,σ_o-essential if there exist a word $v\in\sigma_o\cdot\Sigma_o^*$ and automata $X_I,..,X_{i-I},X_{i+I},..,Y,Z$ such that the output words of the automata $S(F,X_I,..,X_{i-I},Y,X_{i+I},..,X_r)$ and $S(F,X_I,..,X_{i-I},Z,X_{i+I},..,X_r)$ on the word v are different.

Let's fix the letter $\theta\in T$.

Definition. The normal form of the GM-automata $F=\langle\Sigma_o\times\Sigma^r,Q,T_o\times T^r,q_o,\varphi,\psi\rangle$ is the GM-automate $G=\langle\Sigma_o\times\Sigma^r,Q,T_o\times T^r,q_o,\varphi,\mu\rangle$, where $\mu^o(q,\sigma)=\psi^o(q,\sigma)$, and for every $i\in\{I,..,r\}$

$$\mu^{i}(q,\sigma) = \begin{cases} \psi^{i}(q,\sigma), \text{ if } i \text{ is } q,\sigma^{o}\text{-essential channel} \\ \theta \text{ in opposite case} \end{cases}$$

Evidently, the normal form of any GM-automate is a GM-automate.

2.2. Permutations of channels. Definition I. Let $F=\langle\Sigma_0 \times \Sigma^r, Q, T_0 \times T^r, q_0, \varphi, \psi\rangle$ and $G=\langle\Sigma_0 \times \Sigma^r, Q, T_0 \times T^r, q_0, \lambda, \mu\rangle$ be the GM-automata and $(i_I,..,i_r)$ is a permutation.

We say that GM-automate G is obtained from GM-automate F by permutation $(i_I,...,i_r)$ if for every state $q \in Q$, letters $\sigma_0, \sigma_I,.., \sigma_r \in \Sigma_0 \times \Sigma^r$ and $j \in \{I,..,r\}$.

$$\lambda(q, \sigma_0, \sigma_I,.., \sigma_r) = \varphi(q, \sigma_0, \sigma_{i_I},.., \sigma_{i_r})$$
$$\mu^{o}(q, \sigma_0, \sigma_I,.., \sigma_r) = \psi^{o}(q, \sigma_0, \sigma_{i_I},.., \sigma_{i_r})$$
$$\mu^{ij}(q, \sigma_0, \sigma_I,.., \sigma_r) = \psi^{j}(q, \sigma_0, \sigma_{i_I},.., \sigma_{i_r})$$

2. GM-automata F and G are p-equivalent (p-isomorphic) if F is equivalent (isomorphic) to some automate which may be obtained from G by some permutation.

2.3. The minimization theorem. We'll say 'G is a minimal GM-automate for GM-automate F' instead 'G is a GM-automate with smallest number of states which is equivalent to F'.

Lemma I. GM-automata F and G are s-equivalent if and only if their normal forms are p-equivalent.

Using lemma I we can prove the next theorem.

Theorem I. Let F be a finite GM-automate.

I. The reduced automate which is equivalent to normal form of GM-automate F is a minimal GM-automate for F.

2. If G and H are minimal GM-automata for F, then normal forms of G and H are p-equivalent.

Lemma 2. There exist an algorithm which constructs the normal form of the given GM-automate.

Theorem 2. There exist an algorithm of constructing minimal GM-automate for the given finite GM-automate F.

The last theorem immediately follows from theorem I and lemma 2 and from ability to construct reduced automate which is equivalent to the given finite automate.

3. Enriched GM-automata

3.I. Maps connected with GM-automata. Designation. Let Γ and Δ be the finite alphabets. The set of all automata with input alphabet Γ and output alphabet Δ will be denoted $k(\Gamma, \Delta)$.

Every GM-automate F $k(\Sigma_0 \times \Sigma^r, T_0 \times T^r)$ gives a map S_F: $k(T,\Sigma)^r \rightarrow k(\Sigma_0, T_0)$ which maps ordered set $x_I,..,x_r$ into automate $S(F,X_I,..,X_r)$.

Evidently, GM-automata F and G are s-equivalent if and only if maps S_F and S_G are equal.

3.2. Enriched GM-automata and s-distinguishing of them. A map $S_F: \langle x_I,..,x_r\rangle \rightarrow S(F,X_I,..,X_r)$ may be considered as a function of only part of variables $x_I,..,x_r$ treating other variables as parameters.

Except that we may don't want differ maps connected with p-equivalent GM-automata. So we get the next definitions.

Definition. Enriched GM-automate (EGM-automate) is a couple F,f where F is a GM-automate and f is a partial function from $\{I,..\dim F\}$ to the set of integers (about designation dimF, see I.2).

The dimension of EGM-automate $\langle F,f\rangle$ is **dimension** of GM-automate F. Let F,f and G,g be EGM-automata, dim $\langle F,f\rangle = r$, **dim G,g = d**

We'll say that the word v and two sequences of automata $x_I,..x_r$ $y_I,..,y_d$ s-distinguish EGM-automata $\langle F,f\rangle$ and $\langle G,g\rangle$ if (i) for all $i\in\{I,..,r\}$ and $j\in\{I.,.,d\}$ if f(i) and g(j) are defined and $f(i) = g(j)$, then x_i is equivalent to y_j; (ii) output words of automata $S(F,X_I,..,X_r)$ and $S(G,Y_I,..,Y_d)$ on the word v are different.

Definition. EGM-automata $\langle F,f\rangle$ and $\langle G,g\rangle$ are named s-distinguishable if there exist the word v and the sequences $x_I,..,x_r$, $y_I,..,y_d$ which s-distinguish $\langle F,f\rangle$ and $\langle G,g\rangle$.

A connection between s-distinguishing of GM-automata and EGM-automata gives the next lemma.

Lemma 3. Let F and G be the GM-automata, dimF = dimG = r, and E_r is an identical function with domain $\{I,..,r\}$. The word v and automata $x_I,..,x_r$ s-distinguish GM-automata F and G if and only if the word v and the sequences of automata $x_I,..,x_r$, $x_1,...,x_r$ s-distinguish EGM-automata $\langle F, E_r\rangle$ and $\langle G,E_r\rangle$.

3.3. The minimization of EGM-automata. The definitions of normal form of EGM-automate and p-equivalence of EGM-automata are analogous to the correspondent definitions for GM-automata and we won't cite them here.

Definition. EGM-automate $\langle G,g\rangle$ is the minimal EGM-automate for EGM-automate $\langle F,f\rangle$ if $\langle G,g\rangle$ is the EGM-automate with smallest number of states and with smallest dimension, which is not s-distinguishable from $\langle F,f\rangle$.

Theorem 3. Let $\langle F,f\rangle$ be the r-dimensional EGM-automate and f(i) is defined for every $i\in\{I,..,r\}$.

I. There exist an algorithm which constructs the minimal EGM-automate for $\langle F,f\rangle$.

2. If $\langle G,g\rangle$ and $\langle H,h\rangle$ are the minimal EGM-automata for $\langle F,f\rangle$, then the normal forms of $\langle G,g\rangle$ and $\langle H,h\rangle$ are p-equivalent.

4. Length of experiments with schemata

4.I. The main theorems.

Theorem 4. Let $\langle F,f\rangle$ and $\langle G,g\rangle$ be the s-distinguishable EGM-automata, F and G have no more than n states. Let t be a number of such integers i that f(i) = g(j) for some j.

There exist the word v and two sequences of automata $x_I,..,x_r$, $y_I,..,y_d$ such that

(i) v, $x_I,..,x_r$, $y_I,..,y_d$ s-distinguish $\langle F,f\rangle$ and $\langle G,g\rangle$,

(ii) a length of v is no more than $(t+2)\cdot n$.

Next theorem 5 is a simple corollary from lemma 3 and theorem 4.

Theorem 5. Let F and G be r-dimensional non s-equivalent GM-automata, F and G have no more than n states.

There exist such word v and automata $x_I,..,x_r$ that

(i) v and $x_I,..,x_r$ s-distinguish F and G,

(ii) a length of v is no more than $(r+2)\cdot n$.

4.2. Given upper bounds are asymptotically exact.

Theorem 6. There exist non s-equivalent GM-automata F and G such that

(i) dimF = dimG = r,

(ii) F and G have no more than n states,

(iii) For any word v and any automata $x_I,..,x_r$ if the length of v is less than $(n-r+I)\cdot(r+2) - I$, then $v,x_I,..,x_r$ don't s-distinguish F and G.

Literature

I. Kobrinsky, Trakhtenbrot. Introduction to the finite automata theory, Moscow, I962.